

P425/1
PURE MATHEMATICS
PAPER ONE
3 HOURS

NTARE SCHOOL
UGANDA ADVANCED CERTIFICATE OF EDUCATION
PURE MATHEMATICS
PAPER 1
3 HOURS

INSTRUCTIONS TO CANDIDATES

- Answer all eight questions in section A and only five from section B
- All necessary working must be clearly shown.
- Silent, non-programmable scientific calculators and mathematical tables with a list of formula may be used.

SECTION A (40 MARKS)

- The difference between the roots of the equation $3x^2 + ax + 3 = 0$ is $\frac{8}{3}$. Find the possible values of a . (05mks)
- A, B, and C are angles of a triangle and such that $\cos A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$, without using tables or a calculator, show that $\cos C = \frac{33}{65}$. (05mks)
- Use Maclaurin's theorem to expand $\ln \sqrt{1-2x}$ up to the term in x^3 . (05mks)
- Solve for x if $e^x = 1 + 6e^{-x}$. (05mks)
- Use a suitable substitution to evaluate $\int_0^2 \frac{x^2}{(x^3+1)^{3/2}} dx$. (05mk)
- Find the angle between the lines $ax + by + c = 0$ and $(a-b)x + (a+b)y + d = 0$. (05mks)
- Find the perpendicular distance from the point $P(1, -1, 4)$ to the line $r = i + 2j + \mu(2i + j + 2k)$. (05mks)
- Find the area enclosed by the curve $y = 4 - x^2$ and x -axis. (05mks)

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3. Use Maclaurin's theorem to expand $\ln \sqrt{1-2x}$ up to the term in x^3 . (05mks)
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8. Find the area enclosed by the curve $y = 4 - x^2$ and x - axis. (05mks)

$$\begin{array}{r} 111^2 - 62 - 13 \\ 112^2 - 662 + 143 \\ \hline 602 - 156 \end{array}$$

$$\frac{602}{2}$$

SECTION B (60 marks)

✓9. a) Show that $z = -3 - 2i$ is a root of the equation $z^3 + 5z^2 + 7z - 13 = 0$, hence find other roots of the equation. (06mks)

b) Use De Moivre's theorem to show that the expression $(1 + i)^n + (1 - i)^n$ is always real for all values of n , hence find the value of the expression when $n = 8$. (06mks)

10. By substituting $t = e^x$, show that

$$\int_0^{\ln 4} \frac{e^{2x}}{e^{2x} + 3e^x + 2} dx = \ln\left(\frac{8}{5}\right) \quad (12\text{mks})$$

✓11 a) Express $\sqrt{6} \cos \theta + \sqrt{10} \sin \theta$ in the form $R \cos(\theta - \alpha)$ where $R > 0$ and $0^\circ < \alpha < 90^\circ$

hence solve the equation $\sqrt{6} \cos\left(\frac{\theta}{2}\right) + \sqrt{10} \sin\left(\frac{\theta}{2}\right) = 3$ for $0^\circ \leq \theta \leq 360^\circ$. (06mks)

b) If $t = \tan 2\theta$; find $\tan 4\theta$ and $\cos 4\theta$ in terms of t , hence or otherwise prove that

$$\sec 4\theta + \tan 4\theta = \frac{1 + \tan 2\theta}{1 - \tan 2\theta} \quad (06\text{mks})$$

12. a) Show that the curve $x^2 - 6y - 8y + 1 = 0$ represents a parabola. State its vertex. (05mks)

b) The tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $P(a \cos \theta, b \sin \theta)$ meets the x -axis at A

and y -axis at B. Prove that the area of triangle AOB formed, where O is the origin is

$$ab \csc 2\theta \quad (07\text{mks})$$

✓13. The plane M has the equation $3x - 12y - 4z = 36$ and the line l has the equation

$$r = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} 9 \\ 4 \\ 1 \end{pmatrix}$$

a) Find the

(i) Point of intersection of the plane M and the line l . (04mks)

(ii) Angle between the plane M and line l . (03mks)



$$1 \times 2$$

$$a \times b = |a| |b| \sin \theta$$

b) A perpendicular line drawn from $P(12, -2, -7)$ meets the plane M at point N. Find the coordinates of N.

✓14. a) An arithmetic progression has n terms. The sum of the first three terms is 15 and sum of the last three terms is 168. If the tenth term is 29. Find the value of n .

b) Assuming that x is so small that terms in x^3 and higher powers are neglected, find a quadratic approximation of $\sqrt{\frac{1+2x}{1-x}}$, hence

State the range of values of x for which the expansion is valid.

*5.a) Use the substitution $X = 4\sin^2\theta$; to show that

$$\int_0^2 \sqrt{\frac{x}{4-x}} dx = \pi - 2$$

b) Given that $y = \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)$; Show that $\frac{dy}{dx} = 1$

✓16.a) Given that $y = Ae^{-x} + Be^{3x}$, form a differential equation by eliminating the constants A and B. State the order of a differential equation.

b) The rate of increase of temperature of the liquid being heated in an oven is proportional to the excess temperature of the oven over that of the liquid. The temperature of the oven is maintained at 200°C . The temperature of the liquid rises from 0°C to 100° in 5 minutes.

Find the time, it takes the temperature of the liquid to rise to 160°C .

$$(3n-8)$$

$$-3n+9n+24$$

$$(2a + (n-1)d)$$

END

$$S_3 = \frac{3}{2}(2a + 2d) = 15$$

S 6 CHEMISTRY PAPER 2

2019

Time: 2½ hrs

Answer 5 questions including 3 questions from section A and 2 questions from section B

SECTION A. Answer 3 questions only

1. a)i) Write the electronic configuration of chromium (Cr = 24) (1mk)

ii) State the most common oxidation states of chromium (1½ mks)

b) Describe the reaction of chromium with

i) air (3½ mks)

ii) water (2mks)

iii) sulphuric acid (4mks)

c) A solution of sodium hydroxide was added to a solution of chromium (iii) chloride. Explain what was observed and write equations for the reactions that take place (3mks)

d) Hydrogen peroxide was added to the solution formed in (c) above.

i) Explain what was observed (1mk)

ii) Write the equation for the reaction that took place (1mk)

e) Write the equation for the reaction that took place when

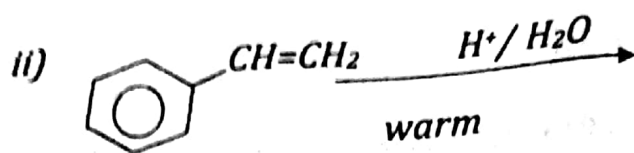
i) dilute hydrochloric acid is added to a solution of potassium chromate (1mk)

ii) hydrogen peroxide is added to acidified potassium dichromate (1mk)

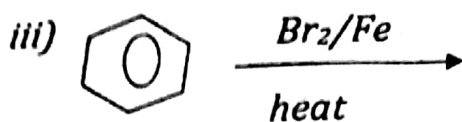
iii) potassium iodide solution is added to acidified potassium dichromate (1mk)

2. a) Complete the following equations and outline the mechanisms

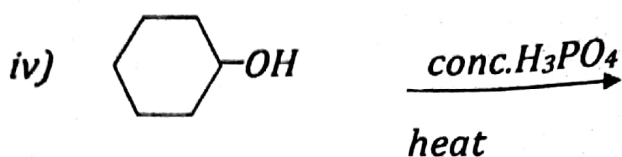
i) $\text{CH}_3\text{CH}_2\text{Br} \xrightarrow{\text{KOH(aq) / heat}}$ (2mks)



(3mks)



(3½mks)



(3½mks)

b) Using relevant examples distinguish between the following mechanisms

i) electrophilic addition and nucleophilic addition

(4mks)

ii) electrophilic substitution and nucleophilic substitution

(4mks)

iii) E_1 and E_2

3. a) i) State Raoult's law and mention its limitations

(1mk)

ii) Explain why some liquid mixture do not obey Raoult's law.

(2mks)

b) Hydrochloric acid and water form an azeotropic mixture which boils at 110°C and contains 20% hydrochloric acid.

i) Define an azeotropic mixture

(1mk)

ii) Explain why the mixture of hydrochloric acid and water deviates from Raoult's law.

(2mks)

c) (i) Sketch a labeled temperature-composition diagram for the mixture of hydrochloric acid and water.

(2mks)

(ii) Using the diagram, describe what happens when a mixture containing 10% hydrochloric acid is distilled

(2mks)

e) The density of the azeotropic mixture of hydrochloric acid and water is 1.18 g cm^{-3} . Calculate

i) The molarity of the acid

(1½ mks)

ii) volume of the acid required to prepare 250 cm^3 of 2M hydrochloric acid (1½

ii) State one similarity and one difference between an azeotropic mixture and a compound. (2 mks)

f) A solvent Y of molecular mass 62 has a vapour pressure of $1.0 \times 10^4 \text{ Nm}^{-2}$ at 298K. 23.3g of a non-volatile solute Z of molecular mass 270 was dissolved in 100g of Y at 298K.

i) Calculate the vapour pressure of the solution.

(2 mks)

ii) State and explain the effect of the concentration of the solute on the boiling point of Y. (3 mks)

4.a) Define

i) conductivity

(1mk)

ii) molar conductivity

(1mk)

b) The table below shows the molar conductivities of sodium hydroxide solutions of different concentrations

Concentration/mol dm^{-3}	0.01	0.04	0.09	0.16	0.25	0.36
Molar conductivity (Λ) $\Omega^{-1} \text{ cm}^2 \text{ mol}^{-1}$	238	230	224	217	210	202

i) Plot a graph of molar conductivity against the square root of concentration (3

ii) Explain the shape of the curve

(3 mks)

iii) Determine the molar conductivity of sodium hydroxide at infinite dilution.

(1mk)

c) The resistance of 0.1M potassium chloride and 0.1M bromoethanoic acid is 24.96 and 66.50 ohms respectively at 25°C. If the conductivity of potassium chloride at 25°C is $0.01164 \text{ ohm}^{-1}\text{cm}^{-1}$ and the molar conductivity of bromoethanoic acid at infinite dilution is $389 \text{ } \Omega^{-1}\text{cm}^2 \text{ mol}^{-1}$, calculate the

i) cell constant (1½ mks)

ii) molar conductivity of bromoethanoic acid (2½ mks)

iii) PH of bromoethanoic acid (1½ mks)

iv) ionisation constant of bromoethanoic acid (3mks)

d). The ionic conductivities of Na^+ and OH^- at infinite dilution are 350 and $198 \text{ scm}^2 \text{ mol}^{-1}$ respectively. Calculate the electrolytic conductivity of 0.2M sodium hydroxide (2½ mks)

SECTION B Answer 2 questions

5. Explain the following observations. Write equations for the reactions that take place where necessary

a) When chloroethane is heated with sodium hydroxide solution, the product acidified with dilute nitric acid followed by silver nitrate, a white precipitate is formed. When chlorobenzene is treated the same way, there is no observable change. (4½ mks)

b) When a concentrated solution of sodium carbonate is added to a solution of potassium chromium (iii) sulphate ($\text{K}_2\text{SO}_4 \cdot \text{Cr}_2(\text{SO}_4)_3 \cdot 24\text{H}_2\text{O}$), a green precipitate is formed with bubbles of a colourless gas. (4½ mks)

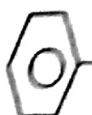
c) The acid ionisation constant of methanoic acid is 4.4×10^{-4} whereas that of ethanoic acid is 1.8×10^{-5} at 25°C. (3mks)

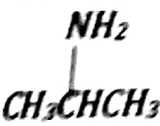
d) Ethoxyethane boils at 35°C whereas butan-1-ol boils at 124°C (4mks)

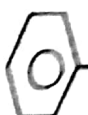
e) A solution of sodium thiosulphate becomes cloudy when it is exposed to air for (2mks)

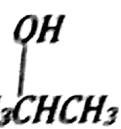
6) Write equations to show how the following compounds can be synthesised. Indicate the reagents and conditions

a) $\text{CH}_3\text{CH}_2\text{CH}_2\text{OH}$ from Chloroethane (3mks)

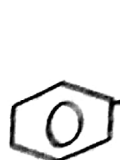
b)  CH_2OH from Benzene (3mks)

c)  from Propyne (3½ mks)

d)  CH=NOH from Benzene (3mks)

e)  from $\text{CH}_3\text{CH}_2\text{CH}_2\text{Cl}$ (3mks)

f) $(\text{CH}_3\text{CH}_2)_2\text{O}$ from $\text{CH}_2=\text{CH}_2$ (2mks)

g)  from benzene (2½ mks)

7) (a) 25 cm³ of 0.1M ammonia solution was titrated with hydrochloric acid and the pH of the solution was measured at intervals. The table below gives the pH of the solution when measured volumes of hydrochloric acid were added

Volume of hydrochloric acid (cm ³)	10.0	15.0	16.5	17.0	20.0	25.0
pH of the solution	9.08	8.30	6.70	2.97	1.96	1.60

(i) Plot a graph of pH against volume of hydrochloric acid

(3mks)

(4mks)

ii) Explain the shape of the graph

(2mks)

iii) Calculate the molarity of hydrochloric acid

b) The PH ranges of some indicators are shown below

Indicator	pH range
Thymol blue	1.2 - 2.8
Methyl red	4.8 - 6.0
Phenolphalein	6.6 - 8.0

Which one of the above indicators is most suitable for the above titration?
Explain your answer.

(1mk)

c) Calculate the

i) molarity of ammonium chloride at the end point

(2mks)

ii) hydrolysis constant of the salt formed

(3mks)

e) 25cm³ of 0.1M ethanoic acid was titrated with sodium hydroxide solution.

i) Draw a sketch graph to show how the pH of the solution changes when sodium hydroxide is added to the acid

(1mk)

e) 20cm³ of 2M sodium hydroxide was added to 100cm³ of 2M ethanoic acid.

Calculate the pH of the solution formed

(4mks)

8. a) i) Describe the industrial manufacture of nitric acid. Write equations for the reactions that take place.

(5marks)

ii) State two uses of nitric acid.

(1mrk)

b) Describe the reaction of nitric acid with

i) copper

(4marks)

ii) sulphur

(2marks)

c) 2.0g of a nitrogenous fertilizer was boiled with excess sodium hydroxide solution.

The ammonia formed reacted with 200cm³ of 0.1M hydrochloric acid.

Calculate the percentage by mass of nitrogen in the fertilizer. (3marks)

d) Name the reagent that you would use to distinguish between nitrate and nitrite ions. State what is observed in each case. (2marks)

e) Write equations for the reactions that take place sodium nitrite solution is added to acidified potassium dichromate(VI) solution. (1mks)

f) (i) State what is observed when zinc nitrate is heated. (1mk)

ii) Write the equation for the reaction that took place (1mk)

UGANDA ADVANCED CERTIFICATE OF EDUCATION

RESOURCEFUL EXAMINATION 2019

P425/2 APPLIED MATHEMATICS

TIME: 3 HOURS

Instructions

Attempt all questions in section A and any five from Section B.

In numerical work, take $g = 9.8 \text{ ms}^{-2}$

SECTION A (40 MARKS)

1. A particle is projected from a point O with a speed of $20\sqrt{2}$ m/s so as to attain maximum range. Express in vector form its velocity $\mathbf{v}$ and displacement $\mathbf{r}$, from O at any time t seconds. (05 marks)

2. The table below is an extract from tables of sines, $\sin x$.

$x = 10^\circ$	0'	10'	20'	30'	40'
$\sin x$	0.1736	0.1765	0.1794	0.1822	0.1851

Use linear interpolation/extrapolation to estimate;

(i) $\sin 10^\circ 49'$

(ii) $\sin^{-1} 0.1805$

$$\frac{y - 0.1822}{0.0029} \times \frac{10}{10}$$

$$\log 0.1822 = 0.2551$$

$$\log 0.1805 = 0.2541$$

(05 marks)

3. Two events A and B are such that $P(A) = 0.6$, $P(B) = 0.3$ and $P(A \cup B) = 0.8$. Find the;

(i) $P(A \cup B')$ $P(A) + P(A \cap B)$

$$P(A) + P(B) - P(A \cap B)$$

(ii) $P(B' / A')$

(05 marks)

4. ABC is an isosceles triangle, right angled at A with $AB = 2\text{m}$. Forces of 8N , 4N and 6N act along the sides BA , CB and CA respectively. Find the magnitude and direction of the resultant force. (05 marks)

5. The price index of a calculator in 2009 based on 2001 was 115. The price index of the same calculator in 2005 based on 2001 was 80. Calculate the;

(i) Price index of the calculator in 2009 based on 2005.

(ii) Price of the calculator in 2005 if its price in 2009 was shs. 40,000.

(05 marks)

6. Use the trapezium rule with 7 ordinates to estimate $\int_2^8 \sin^2\left(\frac{x}{2}\right) dx$, correct to 3 significant figures. (05 marks)

7. A random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{2}{3}x & ; 0 \leq x \leq 1 \\ \frac{1}{3}(3-x) & ; 1 \leq x \leq 3 \\ 0 & ; \text{Otherwise} \end{cases}$$

Calculate the median of X . (05 marks)

8. A particle is moving with linear simple harmonic motion of amplitude $1.5m$. The speed of the particle is $\sqrt{50} ms^{-1}$ when its displacement from the end point is $1m$. Calculate its maximum acceleration. (05 marks)

SECTION B (60 MARKS)

9. (a) A body moving initially with a velocity u covers a distance x after t seconds. If it moves with a uniform acceleration a , derive an expression relating x, t, u and a . (05 marks)

(b) A train approaching a station does two successive half kilometers in $16s$ and $20s$ respectively. Assuming a uniform retardation, calculate the further distance the train runs before it comes to rest. (07 marks)

10. The numbers x and y are approximations with respective errors Δx and Δy .

(i) Show that the maximum relative error in $\frac{x}{y}$ is given by $\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right|$. (06 marks)

(ii) Hence find the limits within which the exact value of $\frac{1.862}{3.10}$ is expected to lie. Give your answer correct to 3 decimal places. (06 marks)

6. Use the trapezium rule with 7 ordinates to estimate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2\left(\frac{x}{2}\right) dx$, correct to 3

significant figures.

(05 marks)

7. A random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{2}{3}x & ; 0 \leq x \leq 1 \\ \frac{1}{3}(3-x) & ; 1 \leq x \leq 3 \\ 0 & ; \text{Otherwise} \end{cases}$$

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(06 marks)

(ii) Hence find the limits within which the exact value of $\frac{1.862}{3.10}$ is expected to lie. Give your answer correct to 3 decimal places.

(06 marks)

11. A body moving with acceleration $e^{2t}\mathbf{i} - 3\sin 2t\mathbf{j} + 4\cos 2t\mathbf{k}$ is initially located at the point $(1, -2, 2)m$ and has a velocity of $4\mathbf{i} - 2\mathbf{j} + \mathbf{k} \text{ ms}^{-1}$. Find the;

(i) speed of the body when $t = \frac{\pi}{4} \text{ s}$. (06 marks)

(ii) distance of the body from the origin at $t = \frac{\pi}{4} \text{ s}$. (06 marks)

✓ 12. The table below shows the marks obtained by students in Fine Art (x) and

	80	76	96	41	68	31	42	88	68	91
Fine Art	80	76	96	41	68	31	42	88	68	91
Mathematics	43	32	27	64	65	64	65	32	64	43

(a) Draw a scatter diagram for the above data and on it draw a line of best fit.

Use the line of best fit to estimate the mark of a student who scored;

(i) 61 in Mathematics

(ii) 25 in Fine Art (07 marks)

(b) Calculate the rank correlation coefficient between the students' performance in the two subjects and comment on your result at 1% level of significance. (05 marks)

13. An examination question has two parts A and B. The probability of a student getting part A correct is $\frac{2}{3}$. If she gets part A correct, the probability that she

gets part B correct is $\frac{3}{4}$, otherwise it is $\frac{1}{6}$. There are three marks for a correct

solution of part A, two marks for a correct solution of part B and a bonus mark if both parts are correct. Find the probability distribution function for the student's total mark and hence find the expected student's total mark and the variance. (12 marks)

14. (a) A machine manufacturing chalk makes approximately 85% that are within the set tolerance limits. If a random sample of 200 pieces of chalk is taken, find the probability that between 20 and 30 pieces of chalk inclusive will lie outside the set tolerance limits. (06 marks)

